CBSE 2025 EXAMINATIONS
XII MATHEMATICS (041)
Series ZXW4Y ♦ Q.P. Code 65/4/1; 65/4/2; 65/4/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/4/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. The principal value of $\sin^{-1}\left[\sin\left(-\frac{10\pi}{3}\right)\right]$ is

(a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Sol. (c) $\sin^{-1}\left[\sin\left(-\frac{10\pi}{3}\right)\right]=\sin^{-1}\left[\sin\left(-3\pi-\frac{\pi}{3}\right)\right]=\sin^{-1}\left[\sin\frac{\pi}{3}\right]=\frac{\pi}{3}$.

02. If A and B are square matrices of same order such that $AB=A$ and $BA=B$, then A^2+B^2 is equal to

(a) $A+B$ (b) BA (c) $2(A+B)$ (d) $2BA$

Sol. (a) $AB=A \Rightarrow A(BA)=A \Rightarrow (AB)A=A \Rightarrow AA=A$ i.e., $A=A^2$

Also $BA=B \Rightarrow B(AB)=B \Rightarrow (BA)B=B \Rightarrow BB=B$ i.e., $B=B^2$

Now $A^2+B^2=A+B$.

03. For real x, let $f(x)=x^3+5x+1$. Then

(a) f is one-one but not onto on $\mathbb{R}$ (b) f is onto on $\mathbb{R}$ but not one-one

(c) f is one-one and onto on $\mathbb{R}$ (d) f is neither one-one nor onto on $\mathbb{R}$

Sol. (c) Since $f'(x)=3x^2+5>0\ \forall\, x\in\mathbb{R}$ so, $f(x)$ is a strictly increasing function. That means, it must be one-one on $\mathbb{R}$.

Since $y=f(x)=x^3+5x+1$ is continuous and takes all values from $-\infty$ to ∞.

That is, for any $y\in\mathbb{R}$, there exists an $x\in\mathbb{R}$. So, $f(x)$ is onto on $\mathbb{R}$.

04. If $y=\sin^{-1}x$, then $(1-x^2)\frac{d^2y}{dx^2}$ is equal to

(a) $x\dfrac{dy}{dx}$ (b) $-x\dfrac{dy}{dx}$ (c) $x^2\dfrac{dy}{dx}$ (d) $-x^2\dfrac{dy}{dx}$

Sol. (a) $y=\sin^{-1}x \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\dfrac{dy}{dx}=1$

$\Rightarrow \sqrt{1-x^2}\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}\times\dfrac{-2x}{2\sqrt{1-x^2}}=0$

$\Rightarrow (1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=0$

$\therefore (1-x^2)\dfrac{d^2y}{dx^2}=x\dfrac{dy}{dx}$

05. The values of λ so that $f(x)=\sin x-\cos x-\lambda x+C$ decreases for all real values of x are

(a) $1<\lambda<\sqrt{2}$ (b) $\lambda\geq 1$ (c) $\lambda\geq\sqrt{2}$ (d) $\lambda<1$

Sol. (c) For $f(x)=\sin x-\cos x-\lambda x+C$, $f'(x)=\cos x+\sin x-\lambda$

$\Rightarrow f'(x)=\sqrt{2}\left(\dfrac{1}{\sqrt{2}}\cos x+\dfrac{1}{\sqrt{2}}\sin x\right)-\lambda=\sqrt{2}\sin\left(\dfrac{\pi}{4}+x\right)-\lambda$

As the function is decreasing on $x\in\mathbb{R}$ so, $f'(x)=\sqrt{2}\sin\left(\dfrac{\pi}{4}+x\right)-\lambda\leq 0$

$\Rightarrow \lambda\geq\sqrt{2}\sin\left(\dfrac{\pi}{4}+x\right)$ …(i)

We know that for all $x\in\mathbb{R}$, $\sin\left(\dfrac{\pi}{4}+x\right)\in[-1,1]$ i.e., $-\sqrt{2}\leq\sqrt{2}\left(\dfrac{\pi}{4}+x\right)\leq\sqrt{2}$ …(ii)

By (i) and (ii), we conclude that $\lambda\geq\sqrt{2}$.

06. If P is a point on the line segment joining (3, 6, –1) and (6, 2, –2) and y-coordinate of P is 4, then its z-coordinate is

(a) $-\dfrac{3}{2}$ (b) 0 (c) 1 (d) $\dfrac{3}{2}$

Sol. (a) Equation of line passing through (3, 6, –1) and (6, 2, –2) is $\dfrac{x-3}{3}=\dfrac{y-6}{-4}=\dfrac{z+1}{-1}=\lambda$.

The coordinates of any random point *say* P on this line will be $P(3\lambda+3,-4\lambda+6,-\lambda-1)$.

As y-coordinate is 4 i.e., $-4\lambda+6=4$ i.e., $\lambda=\dfrac{1}{2}$.

Therefore, the z-coordinate is $-\lambda-1=-\dfrac{1}{2}-1=-\dfrac{3}{2}$.

07. If M and N are square matrices of order 3 such that $\det(M)=m$ and $MN=mI$, then $\det(N)$ is equal to

(a) –1 (b) 1 (c) $-m^2$ (d) m^2

Sol. (d) Considering $|MN|=|mI|$

$\Rightarrow |M||N|=m^3|I|$

$\Rightarrow m\times|N|=m^3\times 1$

$\therefore |N|=m^2$.

08. If $f(x)=\begin{cases}3x-2, & 0<x\le 1\\ 2x^2+ax, & 1<x<2\end{cases}$ is continuous for $x\in(0,2)$, then a is equal to

(a) –4 (b) $-\frac{7}{2}$ (c) –2 (d) –1

Sol. (d) Since $f(x)$ is continuous for $x\in(0,2)$ so, it must be continuous at all the values of the interval $x\in(0,2)$. Therefore, it must be continuous at $x=1\in(0,2)$.

Further it means, $\lim\limits_{x\to 1^+}f(x)=\lim\limits_{x\to 1^-}f(x)=f(1)$.

That is, $\lim\limits_{x\to 1^+}(2x^2+ax)=3(1)-2$

$\Rightarrow 2(1)^2+a(1)=1$

$\therefore a=1-2=-1$.

09. If $f:N\to W$ is defined as $f(n)=\begin{cases}\frac{n}{2}, & \text{if } n \text{ is even}\\ 0, & \text{if } n \text{ is odd}\end{cases}$. Then f is

(a) injective only (b) surjective only
(c) a bijection (d) neither surjective nor injective

Sol. (b) Since $f(1)=f(3)=0$ but, $1\neq 3$. That means, $f(n)$ will not be injective.

Moreover, for each output $f(n)\in W$ (codomain), we have an input $n\in N$ (domain).

Hence, $f(n)$ is surjective.

10. The matrix $\begin{bmatrix}0 & 1 & -2\\ -1 & 0 & -7\\ 2 & 7 & 0\end{bmatrix}$ is a

(a) diagonal matrix (b) symmetric matrix
(c) skew symmetric matrix (d) scalar matrix

Sol. (c) Since $a_{ij}=-a_{ji}$ and the matrix is square matrix also. So, it will be a skew-symmetric matrix.

11. If the sides AB and AC of ΔABC are represented by vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ respectively, then the length of the median through A on BC is

(a) $2\sqrt{2}$ units (b) $\sqrt{18}$ units (c) $\frac{\sqrt{34}}{2}$ units (d) $\frac{\sqrt{48}}{2}$ units

Sol. (c) The median through A will be $\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}=\frac{3\hat{i}+5\hat{k}}{2}$

Required length of the median is $\sqrt{\frac{9}{4}+\frac{25}{4}}=\sqrt{\frac{34}{4}}=\frac{\sqrt{34}}{2}$ units.

12. The function f defined by $f(x)=\begin{cases}x, & \text{if } x\le 1\\ 5, & \text{if } x>1\end{cases}$ is **not** continuous at

(a) $x=0$ (b) $x=1$ (c) $x=2$ (d) $x=5$

Sol. (b) Here $\lim\limits_{x\to 1^+}f(x)=\lim\limits_{x\to 1^+}(5)=5$. Note that, $f(1)=1\neq\lim\limits_{x\to 1^+}f(x)$.

Clearly, f is **not** continuous at $x=1$.

13. If $f(x)=2x+\cos x$, then $f(x)$

(a) has a maxima at $x=\pi$ (b) has a minima at $x=\pi$
(c) is an increasing function (d) is a decreasing function

Sol. (c) For $f(x)=2x+\cos x$, $f'(x)=2-\sin x$

Since $f'(x) > 0$ for all $x \in \mathbb{R}$. [As $-1 \le \sin x \le 1 \ \forall x \in \mathbb{R}$

So, $f(x)$ is an increasing function.

14. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to

(a) $2(\sin x + x\cos\alpha) + C$ (b) $2(\sin x - x\cos\alpha) + C$

(c) $2(\sin x + 2x\cos\alpha) + C$ (d) $2(\sin x + x\sin\alpha) + C$

Sol. (a) $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx = \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos \alpha} dx$

$= \int \frac{2(\cos^2 x - \cos^2 \alpha)}{\cos x - \cos \alpha} dx = 2\int (\cos x + \cos \alpha) dx$

$= 2(\sin x + x\cos\alpha) + C$.

15. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\tan^{-1} e - \frac{\pi}{4}$ (d) $\tan^{-1} e$

Sol. (c) $\int_0^1 \frac{dx}{e^x + e^{-x}} = \int_0^1 \frac{e^x dx}{e^{2x} + 1} = \int_0^1 \frac{e^x dx}{(e^x)^2 + 1^2} = \left[\tan^{-1}(e^x)\right]_0^1 = \tan^{-1}(e) - \tan^{-1} 1 = \tan^{-1}(e) - \frac{\pi}{4}$.

16. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ are

(a) order : 2, degree : 2 (b) order : 2, degree : 1

(c) order : 2, degree : not defined (d) order : 1, degree : not defined

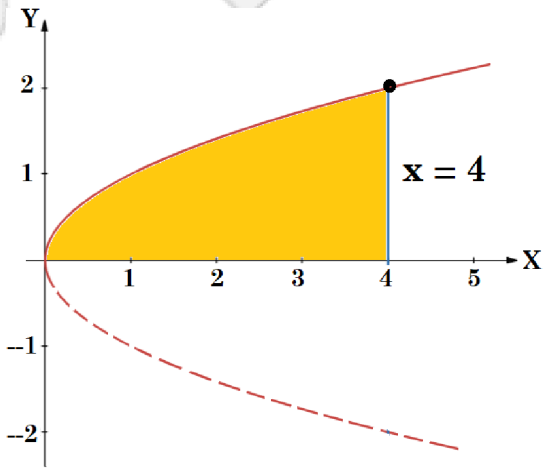
Sol. (c) Order : 2, degree : not defined.

17. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is

(a) $\frac{16}{9}$ Sq.units (b) $\frac{32}{9}$ Sq.units (c) $\frac{16}{3}$ Sq.units (d) $\frac{32}{3}$ Sq.units

Sol. (c) Required area $= \int_0^4 \sqrt{x}\, dx = \frac{2}{3}\left[x^{3/2}\right]_0^4 = \frac{2}{3}\left[4^{3/2} - 0\right] = \frac{16}{3}$ Sq. units.

Refer the diagram shown.



18. The corner points of the feasible region of a Linear Programming are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is

(a) $a = b$ (b) $a = 3b$ (c) $b = 6a$ (d) $3a = 2b$

Sol. (d) As $Z_{(0,2)} = Z_{(3,0)}$ so, $a(0)+b(2)=a(3)+b(0) \Rightarrow 3a=2b$.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** If A and B are two events such that $P(A \cap B)=0$, then A and B are independent events.
Reason (R) : Two events are independent if the occurrence of one does not affect the occurrence of the other.

Sol. (d) Assertion (A) is false but Reason (R) is true.
If $P(A \cap B)=0$, it means the events A and B are mutually exclusive (disjoint) events, not necessarily independent. Two events A and B are independent if $P(A \cap B)=P(A).P(B)$. So, if $P(A \cap B)=0$, then for independence to hold, at least one of P(A) or P(B) must be 0. Otherwise, independence is not guaranteed. Hence, we can conclude that Assertion (A) is false.

20. **Assertion (A) :** In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.
Reason (R) : A feasible region is defined as the region that satisfies all the constraints.

Sol. (a) The statement in Assertion (A) is correct. In a Linear Programming Problem, the feasible region consists of all points that satisfy the given constraints. If the feasible region is empty, it means no point satisfies all the constraints simultaneously, implying that no optimal solution exists. Hence, the LPP has no solution.
Also Reason (R) is correct. Since the feasible region is the set of all points that satisfy the given constraints in an LPP. This definition is correct.
Hence, both Assertion (A) and Reason (R) are true and the Reason (R) correctly explains the Assertion (A), as the definition of feasible region directly supports the claim in Assertion (A).

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. Let A and B be two square matrices of order 3 such that $\det(A)=3$ and $\det(B)=-4$. Find the value of $\det(-6AB)$.

Sol. $|-6AB| = (-6)^3|A||B| = -216\times3\times(-4) = 2592$.
We have used $|kA| = k^n|A|$, where n is order of A; also $|AB| = |A||B|$.

22. (a) Find the least value of 'a' so that $f(x)=2x^2-ax+3$ is an increasing function on $[2, 4]$.

OR

(b) If $f(x)=x+\frac{1}{x}$, $x \geq 1$, show that f is an increasing function.

Sol. (a) For $f(x)=2x^2-ax+3$, $f'(x)=4x-a$
Since f is increasing function on [2, 4] so $f'(x)=4x-a \geq 0 \ \forall \ x \in [2, 4]$
Clearly, minimum value of $f'(x)$ occurs at $x=2$.
It implies, $4(2)-a \geq 0 \Rightarrow a \leq 8$.
Hence, the greatest value of $a=8$.
Therefore, there is 'no least value of a' possible.

OR

(b) For $f(x)=x+\frac{1}{x}$, $f'(x)=1-\frac{1}{x^2}=\left(1-\frac{1}{x}\right)\left(1+\frac{1}{x}\right)$

Note that, $\left(1-\frac{1}{x}\right)\left(1+\frac{1}{x}\right)\geq 0 \ \forall \ x\geq 1$.

That is, $f'(x)\geq 0$ for all $x\geq 1$.

Hence, f is an increasing function.

23. (a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

OR

(b) Find domain of $\sin^{-1}\sqrt{x-1}$.

Sol. (a) Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$ ...(i)

Now $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=\sin^{-1}\left(\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right)=\sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)=\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\times\frac{\cos\theta}{1}\right)$

$=\sin^{-1}(\sin\theta)=\theta=\tan^{-1}x$.

OR

(b) $\sin^{-1}\sqrt{x-1}$ is defined when $-1\leq\sqrt{x-1}\leq 1$

$\Rightarrow 0\leq x-1\leq 1$

$\Rightarrow 1\leq x\leq 2$

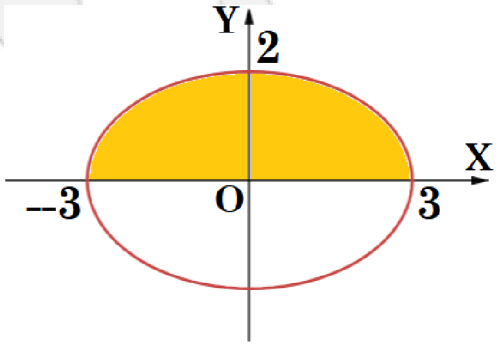
Hence, domain is $x\in[1, 2]$.

24. Calculate the area of the region bounded by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the x-axis using integration.

Sol. From $\frac{x^2}{9}+\frac{y^2}{4}=1$, we get $y^2=\frac{4(9-x^2)}{9}$ i.e., $y=\pm\frac{2}{3}\sqrt{9-x^2}$.

Since the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is symmetrical about both the axes.

So, required area $=2\times\int_0^3\frac{2}{3}\sqrt{9-x^2}\,dx$



$=\frac{4}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$

$=\frac{4}{3}\left[\left(0+\frac{9}{2}\sin^{-1}1\right)-0\right]$

$=\frac{4}{3}\left(\frac{9}{2}\times\frac{\pi}{2}\right)$

$=3\pi$ Sq. units.

25. For the curve $y=5x-2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x=2$?

Sol. Slope of the curve $=\frac{dy}{dx}=5-6x^2=m$

Now $\frac{dm}{dt}=0-12x\frac{dx}{dt}=-12x(2)=-24x$ $\left[\because \frac{dx}{dt}=2 \text{ units/s}\right.$

At $x=2$, $\frac{dm}{dt}=-24(2)=-48$ units/s.

Hence, slope of curve is changing at the rate of 48 units/s.

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. (a) If $f : \mathbb{R}^+ \to \mathbb{R}$ is defined as $f(x) = \log_a x$, $(a > 0$ and $a \neq 1)$, prove that f is a bijection. ($\mathbb{R}^+$ is a set of all positive real numbers.)

OR

(b) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) Write all elements of R.
(ii) Is R a function? Justify.
(iii) Determine domain and range of R.

Sol. (a) Let $x_1, x_2 \in \mathbb{R}^+$ such that $f(x_1) = f(x_2)$.

Then $\log_a x_1 = \log_a x_2 \quad \Rightarrow x_1 = x_2$

So, f is one-one.

Also let $y = f(x), y \in \mathbb{R}$.

That is, $y = \log_a x$

$\Rightarrow x = a^y > 0$ i.e., $x \in \mathbb{R}^+ \ \forall\, y \in \mathbb{R}$.

That is, for each element in codomain $(\mathbb{R})$, we have a pre-image in the domain $(\mathbb{R}^+)$.

Therefore, range = codomain. So, f is onto.

Hence, f is bijection as it is both one-one and onto.

OR

(b) We have $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$.

Also, $R : A \to B$, $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) $R = \{(1, 5), (2, 4)\}$.

(ii) Note that, the element $3 \in A$ has no image in B under R. So, R is not a function.
Note that, a function must assign every element in A to exactly one element in B.

(iii) Domain of $R = \{1, 2\}$, range of $R = \{4, 5\}$.

27. (a) Find k so that $f(x) = \begin{cases} \dfrac{x^2 - 2x - 3}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$.

OR

(b) Check the differentiability of function $f(x) = x|x|$ at $x = 0$.

Sol. (a) As f is continuous at $x = -1$ so, $\lim\limits_{x \to -1} f(x) = f(-1)$.

$$\therefore \lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1} = f(-1)$$

$$\Rightarrow \lim_{x \to -1} \frac{(x + 1)(x - 3)}{x + 1} = k$$

$$\Rightarrow k = \lim_{x \to -1} (x - 3) = -1 - 3 = -4$$

Hence, $k = -4$.

OR

(b) $f(x) = x|x| = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$.

$$Lf'(0)=\lim_{x\to0^-}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0^-}\frac{(-x^2)-(0^2)}{x-0}=\lim_{x\to0^-}(-x)=0.$$

$$Rf'(0)=\lim_{x\to0^+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0^+}\frac{(x^2)-(0^2)}{x-0}=\lim_{x\to0^+}(x)=0.$$

As $Lf'(0)=Rf'(0)$ so, f is differentiable at $x=0$.

28. Evaluate $\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$.

Sol. $$\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\int e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)dx=\int e^x\left[\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}+\left(-\cot\frac{x}{2}\right)\right]dx$$

$$=e^x\left(-\cot\frac{x}{2}\right)+C$$

Here $f(x)=-\cot\frac{x}{2}$, $f'(x)=\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$ and we have used $\int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+C$.

Now $$\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\left[e^x\left(-\cot\frac{x}{2}\right)+C\right]_{\frac{\pi}{2}}^{\pi}$$

$$=\left[e^{\pi}\left(-\cot\frac{\pi}{2}\right)+C\right]-\left[e^{\frac{\pi}{2}}\left(-\cot\frac{\pi}{4}\right)+C\right]$$

$$=\left[e^{\pi}\times0+C\right]-\left[e^{\frac{\pi}{2}}\times(-1)+C\right]=e^{\frac{\pi}{2}}.$$

29. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

Sol. (a) Let X represents number of boys. So, values of X are 0, 1, 2, 3.

Let E be the event of boy.

Then $P(E)=\frac{1}{2}$, $P(\bar{E})=\frac{1}{2}$.

Now $P(X=0)=P(\bar{E})P(\bar{E})P(\bar{E})=\frac{1}{8}$, $P(X=1)={}^3C_1P(E)P(\bar{E})P(\bar{E})=\frac{3}{8}$,

$P(X=2)={}^3C_2P(E)P(E)P(\bar{E})=\frac{3}{8}$, $P(X=3)=P(E)P(E)P(E)=\frac{1}{8}$.

The table of probability distribution is given below.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

OR

(b) Here $S=\{HH, HT, TH, TT\}$.

It is given that, $X=$ Number of heads $-$ Number of tails.

Clearly, values of X are 2 – 0, 1 – 1, 1 – 1, 0 – 2 i.e., 2, 0, 0, –2.

Now $P(X=2)=\frac{1}{4}$, $P(X=0)=2\times\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$, $P(X=-2)=\frac{1}{4}$.

The table of probability distribution is given below.

X	2	0	–2
P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$\text{Mean}=\sum X\,P(X)=2\times\frac{1}{4}+0\times\frac{2}{4}+(-2)\times\frac{1}{4}=\frac{2}{4}-\frac{2}{4}=0$.

30. Find the distance of the point (–1, –5, –10) from the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$.

Sol. Let P(–1, –5, –10). Also, let $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ and $\frac{x-4}{5}=\frac{y-1}{2}=z=\mu$.

The coordinates of random points on the respective lines are $A(2\lambda+1,\ 3\lambda+2,\ 4\lambda+3)$ and $B(5\mu+4,\ 2\mu+1,\ \mu)$.

When the lines intersect, then the points A and B must coincide.

That is, $A(2\lambda+1,\ 3\lambda+2,\ 4\lambda+3)=B(5\mu+4,\ 2\mu+1,\ \mu)$

$\therefore\ 2\lambda+1=5\mu+4\Rightarrow 2\lambda-5\mu=3$ ...(i), $3\lambda+2=2\mu+1\Rightarrow 3\lambda-2\mu=-1$ ...(ii), $4\lambda+3=\mu$ ...(iii)

Solving (i) and (iii), we get $2\lambda-5(4\lambda+3)=3$

$\Rightarrow -18\lambda=18$

$\therefore\ \lambda=-1,\ \mu=-1$.

LHS of (ii) : $3\lambda-2\mu=3(-1)-2(-1)=-3+2=-1=$ RHS of (ii).

Hence, the points A and B coincide. So, the point of intersection is $A(-1,\ -1,\ -1)$.

$\therefore$ Required distance $(AP)=\sqrt{(-1+1)^2+(-5+1)^2+(-10+1)^2}=\sqrt{0+16+81}=\sqrt{97}$ units.

31. Solve the following Linear Programming Problem using graphical method.

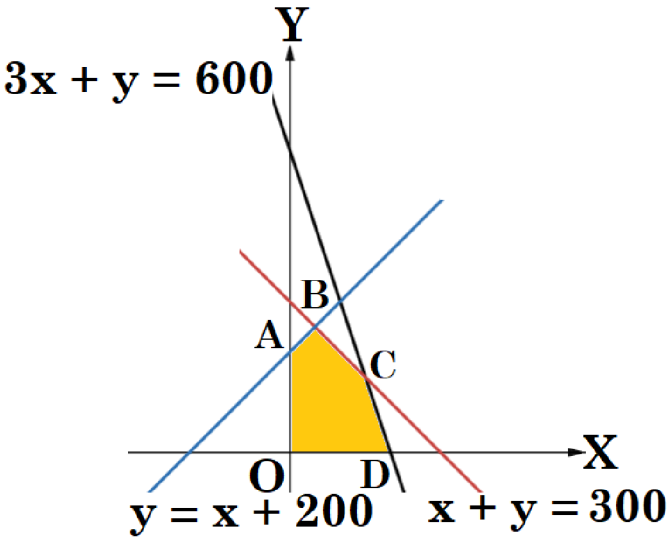
Maximise $Z=100x+50y$

Subject to the constraints $3x+y\le 600,\ x+y\le 300,\ y\le x+200,\ x\ge 0,\ y\ge 0$.

Sol. Consider the graph shown below.

Corner Points	Value of Z
A(0, 200)	10000
B(50, 250)	17500
C(150, 150)	22500 ← Max.
D(200, 0)	20000

Maximum value of $Z=22500$ and it is obtained at (150, 150).



SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. If A is a 3×3 invertible matrix, show that for any scalar $k\neq0$, $(kA)^{-1}=\frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$.

Sol. Let $kA=B$

$\Rightarrow A=\frac{1}{k}B,\ k\neq0$

$\Rightarrow A^{-1}A=\frac{1}{k}A^{-1}B$ [Pre-multiplying by A^{-1} both sides

$\Rightarrow I=\left(\frac{1}{k}A^{-1}\right)B$ [$\because PQ=QP=I\Rightarrow P^{-1}=Q,\ Q^{-1}=P$

$\therefore B^{-1}=\frac{1}{k}A^{-1}$

That is, $(kA)^{-1}=\frac{1}{k}A^{-1}$.

For $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$, $|A|=2(3)-(-1)(-1)+1(-1)=4$ and $\text{adj.}A=\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$.

So, $A^{-1}=\frac{\text{adj.}A}{\det.(A)}=\frac{1}{4}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$.

Hence, $(3A)^{-1}=\frac{1}{3}A^{-1}=\frac{1}{12}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$.

33. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y=4x-\frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight.
(ii) In how many days will the plant attain its maximum height? What is the maximum height?

Sol. (i) $y=4x-\frac{1}{2}x^2\quad\Rightarrow\frac{dy}{dx}=4-\frac{1}{2}\times2x=4-x$

(ii) For $\frac{dy}{dx}=0,\ 4-x=0\quad\Rightarrow x=4$.

Also $\frac{d^2y}{dx^2}=-1<0$ at $x=4$.

So, height of plant (y) will be maximum in 4 days.

Maximum value of height is $=4\times4-\frac{1}{2}\times4^2=16-8=8$ cm.

34. (a) Find $\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}dx$.

OR

(b) Evaluate $\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$.

Sol. (a) Put $\sin x = t \Rightarrow \cos x\, dx = dt$

So, $\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}dx = \int \frac{\cos x}{(4+\sin^2 x)(5-4+4\sin^2 x)}dx$ [$\because \cos^2 x = 1-\sin^2 x$

$$= \int \frac{\cos x}{(4+\sin^2 x)(1+4\sin^2 x)}dx$$

$$= \int \frac{1}{(4+t^2)(1+4t^2)}dt$$

$$= \int \frac{1}{15}\left[\frac{4}{(1+4t^2)} - \frac{1}{(4+t^2)}\right]dt$$

$$= \frac{1}{15}\left[4\times\frac{1}{2}\tan^{-1}(2t) - \frac{1}{2}\tan^{-1}\frac{t}{2}\right] + C$$

$$= \frac{2}{15}\tan^{-1}(2\sin x) - \frac{1}{30}\tan^{-1}\frac{\sin x}{2} + C.$$

OR

(b) Let $f(x) = \frac{1}{a^2\cos^2 x + b^2\sin^2 x}$.

As $f(\pi - x) = \frac{1}{a^2\cos^2(\pi-x) + b^2\sin^2(\pi-x)} = \frac{1}{a^2\cos^2 x + b^2\sin^2 x} = f(x)$.

Using $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$, if $f(2a-x) = f(x)$, we get

$$\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = 2\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$ [Divide Nr and Dr by $\cos^2 x$

$$= 2\int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2 + b^2\tan^2 x}$$ [Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

When $x = 0 \Rightarrow t = 0$, when $x \to \frac{\pi}{2} \Rightarrow t \to \infty$]

$$= 2\int_0^{\infty} \frac{dt}{a^2 + b^2 t^2} = 2\left[\frac{1}{a}\times\frac{1}{b}\tan^{-1}\frac{bt}{a}\right]_0^{\infty}$$

$$= \frac{2}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{ab}.$$

35. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a}\times\vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

OR

(b) Find the equation of a line in vector and Cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and

$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

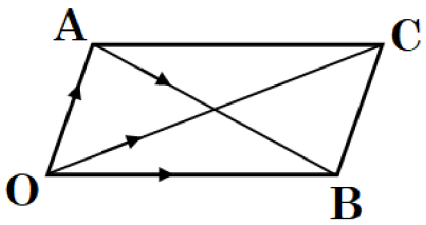
Sol. (a) Consider the diagram shown below.

Assume that $\overrightarrow{OA} = \vec{p}$ and $\overrightarrow{OB} = \vec{q}$ represent the adjacent sides of parallelogram.

It is clear that, $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{OA} = \vec{q} + \vec{p} = \vec{a}$ …(i)

Also, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{q} - \vec{p} = \vec{b}$ …(ii)

[Using triangular law of vectors



By (i) and (ii), $\vec{p} = \frac{\vec{a} - \vec{b}}{2}$, $\vec{q} = \frac{\vec{a} + \vec{b}}{2}$.

Now, the area of parallelogram $= |\vec{p} \times \vec{q}| = \left|\frac{\vec{a} - \vec{b}}{2} \times \frac{\vec{a} + \vec{b}}{2}\right| = \frac{1}{4}|\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}|$

$= \frac{1}{4}|\vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0}|$

$= \frac{1}{4}|2\vec{a} \times \vec{b}|$

$= \frac{1}{2}|\vec{a} \times \vec{b}|$.

Consider $(2\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} + 3\hat{j} - \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{i} + 3\hat{j} + 7\hat{k}$

$\therefore$ Required area of the parallelogram $= \frac{1}{2}|-2\hat{i} + 3\hat{j} + 7\hat{k}| = \frac{\sqrt{4+9+49}}{2} = \frac{\sqrt{62}}{2}$ Sq. units.

OR

(b) Let the d.r.'s of required line passing through (1, 2, –4) be a, b, c.

So, the required line is $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ ...(i)

Since line (i) is perpendicular to $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

So by using $a_1a_2 + b_1b_2 + c_1c_2 = 0$, we get

$3a - 16b + 7c = 0$ …(ii) and $3a + 8b - 5c = 0$ …(iii)

Solving (ii) and (iii), we get $c = 2b$, $a = \frac{2b}{3}$.

Therefore, equation of line (i) becomes : $\frac{x-1}{\frac{2b}{3}} = \frac{y-2}{b} = \frac{z+4}{2b}$ i.e., $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$; this is the Cartesian equation of line.

Also the vector equation of line is $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. Some students are having a misconception while comparing decimals. For example, a student may mention that $78.56 > 78.9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question.

In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table.

Name of student	Distance of javelin (in meters)
Ajay	47.7
Bijoy	47.07
Kartik	43.09
Dinesh	43.9
Devesh	45.2

The students were asked to identify who has thrown the javelin the farthest.
Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students, who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions.

(i) What is the probability of a student not having misconception but still answers Bijoy in the test?

(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test?

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception?

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception?

Sol. Let E_1 and E_2 denote the events of the students who have misconception and who do not have the misconception. Also let E be the event that students answered Bijoy as the correct answer.

Now $P(E_1) = 40\%,\ P(E_2) = 60\%,\ P(E \mid E_1) = 80\%,\ P(\bar{E} \mid E_2) = 90\%$.

(i) As $P(E \mid E_2) + P(\bar{E} \mid E_2) = 1 \quad \Rightarrow P(E \mid E_2) + 90\% = 1$

$\therefore P(E \mid E_2) = 10\% = \frac{1}{10} = 0.1$.

(ii) By using law of total probability, we get $P(E) = P(E_1) \times P(E \mid E_1) + P(E_2) \times P(E \mid E_2)$

$\Rightarrow P(E) = \frac{40}{100} \times \frac{80}{100} + \frac{60}{100} \times \frac{10}{100} = \frac{32+6}{100} = \frac{19}{50} = 0.38 = 38\%$.

(iii) (a) By using Bayes' theorem, $P(E_1 \mid E) = \frac{P(E_1) \times P(E \mid E_1)}{P(E)}$

$\Rightarrow P(E_1 \mid E) = \frac{\frac{40}{100} \times \frac{80}{100}}{\frac{38}{100}} = \frac{32}{38} = \frac{16}{19}$.

OR

(iii) (b) By using Bayes' theorem, $P(E_2 \mid E) = \frac{P(E_2) \times P(E \mid E_2)}{P(E)}$

$\Rightarrow P(E_2 \mid E) = \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{38}{100}} = \frac{6}{38} = \frac{3}{19}$.

37. An engineer is designing a new metro rail network in a city.



Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions.

(i) Find whether the two metro tracks are parallel.

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1,-2,-3)$.

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway.

OR

(iii) (b) Find the shortest distance between Line A and Line B.

Sol. **(i)** The d.r.'s of l_1 and l_2 are respectively 3, –2, 4 and 2, 1, –3.

Since the d.r.'s are not proportional so, the lines are not parallel. Note that $\frac{3}{2} \neq \frac{-2}{1} \neq \frac{4}{-3}$.

Hence the two metro tracks are not parallel.

(ii) Required equation of line : $\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z+3}{4}$.

As the parallel lines have proportionate direction ratios and required line is parallel to line l_1.

(iii) (a) Let the direction ratios be a, b, c for the equation of the pedestrian walkway.

Therefore, its equation is $\frac{x-3}{a} = \frac{y-2}{b} = \frac{z-1}{c}$ …(i)

Since it is given that (i) is perpendicular to both the lines l_1 and l_2.

So, using $a_1a_2 + b_1b_2 + c_1c_2 = 0$, we get $3a - 2b + 4c = 0$, $2a + b - 3c = 0$

Solving these equations, we get $a = \frac{2c}{7}$, $b = \frac{17c}{7}$

By (i), required line is $\frac{x-3}{\frac{2c}{7}} = \frac{y-2}{\frac{17c}{7}} = \frac{z-1}{c}$ i.e., $\frac{x-3}{2} = \frac{y-2}{17} = \frac{z-1}{7}$.

OR

(iii) (b) For the given lines l_1 and l_2, we have

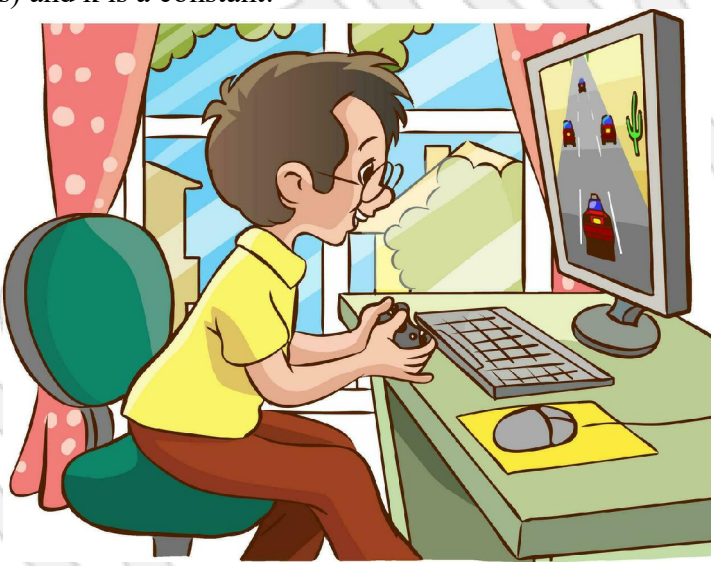
$\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$; $\vec{a}_2 = \hat{i} + 3\hat{j} - 2\hat{k}$ $\vec{b}_2 = 2\hat{i} + \hat{j} - 3\hat{k}$.

Now $\vec{a}_2 - \vec{a}_1 = -\hat{i} + 4\hat{j} - 5\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = 2\hat{i} + 17\hat{j} + 7\hat{k}$

$$\text{Shortest distance} = \frac{\left|\vec{b}_1 \times \vec{b}_2 . (\vec{a}_2 - \vec{a}_1)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|} = \frac{\left|(2\hat{i} + 17\hat{j} + 7\hat{k}).(-\hat{i} + 4\hat{j} - 5\hat{k})\right|}{\left|2\hat{i} + 17\hat{j} + 7\hat{k}\right|}$$

$$= \frac{|-2 + 68 - 35|}{\sqrt{4 + 289 + 49}} = \frac{31}{\sqrt{342}} = \frac{31}{3\sqrt{38}} \text{ units}.$$

38. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature ($25^\circ C$). Initially the processor's temperature is $85^\circ C$. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$, where $T(t)$ represents the temperature of the processor at time t (in minutes) and k is a constant.



Based on the above information, answer the following questions.

(i) Find the expression for temperature of processor $T(t)$, given that $T(0) = 85^\circ C$.

(ii) How long will it take for the processor's temperature to reach $40^\circ C$?

Given that $k = 0.03$, $\log_e 4 = 1.3863$.

Sol. **(i)** Given $\frac{d}{dt}(T(t)) = -k(T(t) - 25) \Rightarrow \frac{d(T(t))}{(T(t) - 25)} = -k\,dt$

On integrating, we get $\int \frac{d(T(t))}{(T(t) - 25)} = \int -k\,dt$

$\Rightarrow \log|(T(t) - 25)| = -kt + \lambda$, where λ is constant of integration

$\Rightarrow (T(t) - 25) = e^{-kt + \lambda}$ i.e., $T(t) = Ce^{-kt} + 25$, where $C = e^{\lambda}$.

As it is given that $T(0) = 85^\circ C$, so $T(0) = Ce^{-k \times 0} + 25$

$\Rightarrow 85 = C \times e^0 + 25 \quad \Rightarrow C = 85 - 25 = 60$

$\therefore$ Required expression is $T(t) = 60\,e^{-kt} + 25$ i.e., $T(t) = 60\,e^{-0.03t} + 25$.

(ii) When $T(t) = 40^\circ C$, then $40 = 60\,e^{-0.03t} + 25$

$\Rightarrow 15 = 60\,e^{-0.03t}$

$\Rightarrow e^{0.03t} = 4$

Taking log on both the sides, we get $\log e^{0.03t} = \log 4$

$\Rightarrow 0.03t \times \log e = 1.3863 \qquad [\because \log e = 1$

$\Rightarrow t = \dfrac{1.3863}{0.03} = 46.21$ minutes.

☑ Q.P. Code - 65/4/2

01. The principal branch of $\cos^{-1} x$ is

(a) $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$ (b) $[\pi, 2\pi]$ (c) $[0, \pi]$ (d) $[2\pi, 3\pi]$

Sol. (c) The principal branch of $\cos^{-1} x$ is $[0, \pi]$.

04. If $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{for } x \neq \dfrac{\pi}{2} \\ k, & \text{for } x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \dfrac{\pi}{2}$, then the value of k is

(a) $\dfrac{3}{2}$ (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{2}$ (d) 1

Sol. (c) As f is continuous at $x = \dfrac{\pi}{2}$ so, $\lim\limits_{x \to \frac{\pi}{2}} f(x) = f\left(\dfrac{\pi}{2}\right)$.

That is, $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1-\sin^3 x}{3\cos^2 x} = k$

$\Rightarrow \lim\limits_{x \to \frac{\pi}{2}} \dfrac{(1-\sin x)(1+\sin x+\sin^2 x)}{3(1-\sin^2 x)} = k$

$\Rightarrow \lim\limits_{x \to \frac{\pi}{2}} \dfrac{(1-\sin x)(1+\sin x+\sin^2 x)}{3(1-\sin x)(1+\sin x)} = k$

$\Rightarrow k = \lim\limits_{x \to \frac{\pi}{2}} \dfrac{(1+\sin x+\sin^2 x)}{3(1+\sin x)}$

$\Rightarrow k = \dfrac{\left(1+\sin\dfrac{\pi}{2}+\sin^2\dfrac{\pi}{2}\right)}{3\left(1+\sin\dfrac{\pi}{2}\right)} = \dfrac{1+1+1^2}{3(1+1)} = \dfrac{1}{2}.$

06. If the direction cosines of a line are $\lambda, \lambda, \lambda$, then λ is equal to

(a) $-\dfrac{1}{\sqrt{3}}$ (b) 1 (c) $\dfrac{1}{\sqrt{3}}$ (d) $\pm\dfrac{1}{\sqrt{3}}$

Sol. (d) Using $l^2 + m^2 + n^2 = 1$, we get $\lambda^2 + \lambda^2 + \lambda^2 = 1 \quad \Rightarrow 3\lambda^2 = 1$

$\therefore \lambda = \pm\frac{1}{\sqrt{3}}$.

07. If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57$, the product of the possible values of x is

(a) –24 (b) –16 (c) 16 (d) 24

Sol. (a) Expanding along 1st column, we get $-1(3x^2-3)-1(6x-12)+0=-57$

$\Rightarrow -3x^2-6x+72=0 \Rightarrow x^2+2x-24=0$

Product of roots of a quadratic equation $= \frac{\text{constant}}{\text{coefficient of } x^2} = \frac{-24}{1} = -24$.

14. $\int e^x(\cos x - \sin x)dx$ is equal to

(a) $e^x \sin x + C$ (b) $-e^x \sin x + C$ (c) $-e^x \cos x + C$ (d) $e^x \cos x + C$

Sol. (d) Using $\int e^x[f(x)+f'(x)]dx = f(x)e^x + C$, here $f(x)=\cos x$, $f'(x)=-\sin x$, we get

$\int e^x(\cos x - \sin x)dx = e^x \cos x + C$.

16. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is

(a) $-\cos x$ (b) $\sec x$ (c) $\log \sec x$ (d) $e^{\sec x}$

Sol. (b) For $\frac{dy}{dx} + y\tan x = \sec x$, $P(x)=\tan x$. So, integrating factor $= e^{\int \tan x\,dx} = e^{\log \sec x} = \sec x$.

21. Using matrices and determinants, find the value (s) of k for which the pair of equations $5x-ky=2$, $7x-5y=3$ has a unique solution.

Sol. Let $A = \begin{bmatrix} 5 & -k \\ 7 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

As the pair of equations has a unique solution, so $|A| = \begin{vmatrix} 5 & -k \\ 7 & -5 \end{vmatrix} \neq 0$

$\Rightarrow -25-(-7k) \neq 0$

$\therefore k \neq \frac{25}{7}$.

25. Find the local maxima and local minima of the function $f(x) = \frac{8}{3}x^3 - 12x^2 + 18x + 5$.

Sol. For $f(x) = \frac{8}{3}x^3 - 12x^2 + 18x + 5$, $f'(x) = 8x^2 - 24x + 18 = 2(4x^2 - 12x + 9)$

$\Rightarrow f'(x) = 2(2x-3)^2$

For $f'(x)=0$, $2(2x-3)^2 = 0 \Rightarrow x = \frac{3}{2}$

Note that, $f'(x)$ does not change its sign as x passes through $x = \frac{3}{2}$.

In fact, in the left and right neighborhood of $x = \frac{3}{2}$, sign of $f'(x)$ is positive.

That means, $x = \frac{3}{2}$ is neither the point of local maxima nor of local minima.

Hence, there is no local maxima and local minima of the function $f(x)$.

28. Find $\int \frac{\cos x\,dx}{1+\cos x+\sin x}$.

Sol. Let $\cos x = A(1+\cos x+\sin x)+B\frac{d}{dx}(1+\cos x+\sin x)+C$

$\Rightarrow \cos x = A(1+\cos x+\sin x)+B(\cos x-\sin x)+C$

On comparing the coefficients of like terms, we get $A+B=1,\ A-B=0,\ A+C=0$

On solving these equations, we get $A=\frac{1}{2},\ B=\frac{1}{2},\ C=-\frac{1}{2}$.

Now let $I=\int \frac{\cos x\,dx}{1+\cos x+\sin x}$

$$=\frac{1}{2}\int\frac{(1+\cos x+\sin x)}{(1+\cos x+\sin x)}dx+\frac{1}{2}\int\frac{(\cos x-\sin x)}{(1+\cos x+\sin x)}dx-\frac{1}{2}\int\frac{1}{(1+\cos x+\sin x)}dx$$

$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{(\cos x-\sin x)}{(1+\cos x+\sin x)}dx-\frac{1}{2}\int\frac{1}{1+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}dx$$

$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{(\cos x-\sin x)}{(1+\cos x+\sin x)}dx-\frac{1}{2}\int\frac{1+\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}+1-\tan^2\frac{x}{2}+2\tan\frac{x}{2}}dx$$

$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{(\cos x-\sin x)}{(1+\cos x+\sin x)}dx-\int\frac{\frac{1}{2}\sec^2\frac{x}{2}}{2\left(1+\tan\frac{x}{2}\right)}dx$$

In second integral, put $(1+\cos x+\sin x)=t \Rightarrow (\cos x-\sin x)dx=dt$.

In third integral, put $1+\tan\frac{x}{2}=u \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx=du$.

$\therefore I=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{dt}{t}-\int\frac{du}{2u}=\frac{1}{2}x+\frac{1}{2}\log|t|-\frac{1}{2}\log|u|+C$

$\Rightarrow I=\frac{x}{2}+\frac{1}{2}\log|1+\cos x+\sin x|-\frac{1}{2}\log\left|1+\tan\frac{x}{2}\right|+C$

Alternatively, let $I=\int\frac{\cos x\,dx}{1+\cos x+\sin x}=\int\frac{\left[\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right]dx}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$

$$\Rightarrow I=\int\frac{\left[\cos\frac{x}{2}-\sin\frac{x}{2}\right]\left[\cos\frac{x}{2}+\sin\frac{x}{2}\right]dx}{2\cos\frac{x}{2}\left[\cos\frac{x}{2}+\cos\frac{x}{2}\right]}$$

$$\Rightarrow I=\int\frac{\left[\cos\frac{x}{2}-\sin\frac{x}{2}\right]dx}{2\cos\frac{x}{2}}=\frac{1}{2}\int\left[1-\tan\frac{x}{2}\right]dx$$

$$\Rightarrow I=\frac{1}{2}\left[x+2\log\left|\cos\frac{x}{2}\right|\right]+C$$

$$\therefore I=\frac{x}{2}+\log\left|\cos\frac{x}{2}\right|+C.$$

29. (a) Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows a tail, then throw a die. Find the conditional probability of the event A : 'the die shows a number greater than 3' given that B : 'there is at least one tail'.

OR

(b) The probability distribution of a random variable X is given as :

X	1	2	3	2λ	3λ	4λ
P(X)	$\frac{11}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{15}$	$\frac{1}{10}$

(i) Calculate λ, if $E(X)=3.2$.

(ii) Find $P(X>1)$.

Sol. (a) Here $S=\{(HH), (HT), (T1), (T2), (T3), (T4), (T5), (T6)\}$.

Let A : the die shows a number greater than 3, B : at least one tail appears on the coin.

So, $A=\{(T4), (T5), (T6)\}$, $B=\{(HT), (T1), \ldots, (T6)\}$ and, $A\cap B=\{(T4), (T5), (T6)\}$.

We have $P(A\cap B)=\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}=3\times\frac{1}{12}=\frac{1}{4}$ and,

$$P(B)=\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}=\frac{3}{4}$$

$$\therefore P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}.$$

OR

(b) (i) $E(X)=\sum X.P(X)=3.2$

$$\Rightarrow 1\times\frac{11}{30}+2\times\frac{1}{15}+3\times\frac{1}{10}+2\lambda\times\frac{3}{10}+3\lambda\times\frac{1}{15}+4\lambda\times\frac{1}{10}=3.2$$

$$\Rightarrow \frac{11}{30}+\frac{4}{30}+\frac{9}{30}+2\lambda\times\frac{9}{30}+3\lambda\times\frac{2}{30}+4\lambda\times\frac{3}{30}=3.2$$

$$\Rightarrow \frac{24}{30}+\lambda\left(\frac{18+6+12}{30}\right)=3.2$$

$$\Rightarrow \frac{24}{30}+\lambda\times\frac{36}{30}=3.2$$

$$\Rightarrow 24+36\lambda=3.2\times30=96$$

$$\Rightarrow 36\lambda=96-24=72$$

$$\therefore \lambda=\frac{72}{36}=2.$$

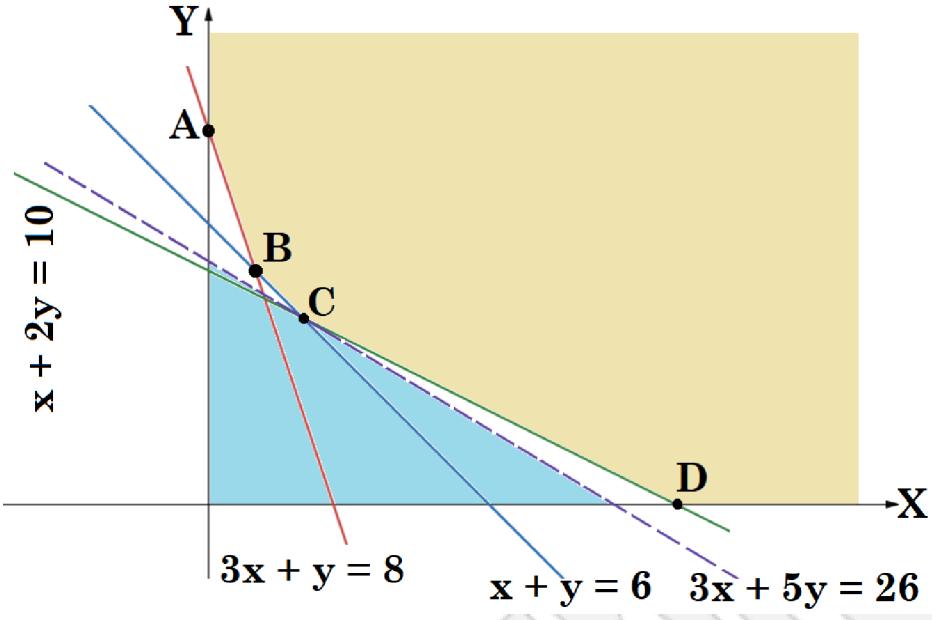
(ii) $P(X>1)=1-P(X\le1)=1-P(X=1)=1-\frac{11}{30}=\frac{19}{30}$.

31. Solve the following Linear Programming Problem graphically.

Minimise $Z=3x+5y$

subject to the constraints $x+2y\ge10$, $x+y\ge6$, $3x+y\ge8$; $x, y\ge0$.

Sol. Consider the graph shown below.



The corner points are A(0, 8), B(1, 5), C(2, 4), D(10, 0).

Corner Points	Value of Z
A(0, 8)	40
B(1, 5)	28
C(2, 4)	26 ← Minimum
D(10, 0)	30

The minimum value of Z is 26. But since the feasible region is unbounded so, $Z = 26$ may or may not be the minimum value of Z. To check, let $3x + 5y < 26$.

As there is no common point between the feasible region and $3x + 5y < 26$. So, $Z = 26$ is the minimum value and it is obtained at (2, 4).

34. (a) Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$.

OR

(b) Find $\int \frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log x\right]}{x^2} dx$.

Sol. (a) Put $\sin^2 x = t \Rightarrow \sin x \cos x \, dx = \frac{dt}{2}$. As $x = 0 \Rightarrow t = 0$; also $x = \frac{\pi}{4} \Rightarrow t = \frac{1}{2}$.

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{(1-\sin^2 x)^2 + \sin^4 x} dx$$

$$\Rightarrow I = \frac{1}{2}\int_0^{\frac{1}{2}} \frac{1}{(1-t)^2 + t^2} dt = \frac{1}{2}\int_0^{\frac{1}{2}} \frac{1}{2t^2 - 2t + 1} dt = \frac{1}{4}\int_0^{\frac{1}{2}} \frac{1}{t^2 - t + \frac{1}{2}} dt$$

$$\Rightarrow I = \frac{1}{4}\int_0^{\frac{1}{2}} \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dt$$

$$\Rightarrow I=\frac{1}{4}\times 2\left[\tan^{-1}(2t-1)\right]_0^{1/2}=\frac{1}{2}\left[\tan^{-1}(0)-\tan^{-1}(-1)\right]$$

$$\Rightarrow I=\frac{1}{2}\left[0+\frac{\pi}{4}\right]=\frac{\pi}{8}.$$

OR

(b) Let $I=\int\sqrt{1+x^2}\,\frac{\left[\log(1+x^2)-2\log x\right]}{x^4}dx$

$$\Rightarrow I=\int\frac{\sqrt{1+x^2}}{x}\times\left[\log(1+x^2)-\log x^2\right]\frac{dx}{x^3}$$

$$\Rightarrow I=\int\sqrt{\frac{1+x^2}{x^2}}\left[\log\left(\frac{1+x^2}{x^2}\right)\right]\frac{dx}{x^3}$$

$$\Rightarrow I=\int\sqrt{1+\frac{1}{x^2}}\left[\log\left(1+\frac{1}{x^2}\right)\right]\frac{dx}{x^3}$$

Put $1+\frac{1}{x^2}=t\Rightarrow\frac{dx}{x^3}=-\frac{dt}{2}$.

$$\therefore I=-\frac{1}{2}\int\sqrt{t}\,\log t\,dt$$

$$\Rightarrow I=-\frac{1}{2}\left\{\log t\int\sqrt{t}\,dt-\int\left(\frac{d}{dt}(\log t)\int\sqrt{t}\,dt\right)dt\right\}$$

$$\Rightarrow I=-\frac{1}{2}\left\{\frac{2t^{3/2}}{3}\log t-\frac{2}{3}\int\left(\frac{1}{t}\times t^{3/2}\right)dt\right\}$$

$$\Rightarrow I=-\left\{\frac{t^{3/2}}{3}\log t-\frac{1}{3}\left(\frac{2t^{3/2}}{3}\right)\right\}+C$$

$$\Rightarrow I=-\frac{t^{3/2}}{3}\left\{\log t-\frac{2}{3}\right\}+C$$

So, $I=-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$.

Remark The integrand given in (b) is wrong, however we have taken correct integrand.

☑ Q.P. Code - 65/4/3

01. Domain of $\sin^{-1}(2x^2-3)$ is

(a) $(-1,\,0)\cup(1,\,\sqrt{2})$ (b) $(-\sqrt{2},\,-1)\cup(0,\,1)$

(c) $[-\sqrt{2},\,-1]\cup[1,\,\sqrt{2}]$ (d) $(-\sqrt{2},\,-1)\cup(1,\,\sqrt{2})$

Sol. (c) $\sin^{-1}(2x^2-3)$ is defined when $-1\le(2x^2-3)\le 1$

That is, $2\le(2x^2)\le 4$

$\Rightarrow 1\le x^2\le 2$

$\Rightarrow -\sqrt{2}\le x\le -1\cup 1\le x\le\sqrt{2}$

Hence, the domain is $x\in[-\sqrt{2},\,-1]\cup[1,\,\sqrt{2}]$.

04. If $y=\log_{2x}(\sqrt{2x})$, then $\frac{dy}{dx}$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{x}$ (d) $\frac{1}{\sqrt{2x}}$

Sol. (a) $y = \log_{2x}(\sqrt{2x}) = \frac{1}{2}\log_{2x}(2x) = \frac{1}{2}\times 1 \quad \therefore \frac{dy}{dx} = 0$.

06. The coordinates of the foot of the perpendicular drawn from the point $A(-2, 3, 5)$ on the y-axis is

(a) $(0, 0, 5)$ (b) $(0, 3, 0)$ (c) $(-2, 0, 5)$ (d) $(-2, 0, 0)$

Sol. (b) $(0, 3, 0)$.

07. If A and B are invertible matrices of order 3×3 such that $\det(A) = 4$ and $\det[(AB)^{-1}] = \frac{1}{20}$, then $\det(B)$ is equal to

(a) $\frac{1}{20}$ (b) $\frac{1}{5}$ (c) 20 (d) 5

Sol. (d) $\left|(AB)^{-1}\right| = \frac{1}{20} \Rightarrow |(AB)|^{-1} = \frac{1}{20} \Rightarrow \left[|A||B|\right]^{-1} = \frac{1}{20}$

$\Rightarrow 4\times|B| = 20$

$\therefore |B| = 5$.

14. If $\int e^{-3\log x}dx = f(x) + C$, then $f(x)$ is

(a) $e^{-3\log x}$ (b) $e^{\log\left(\frac{1}{x^3}\right)}$ (c) $-\frac{1}{2x^2}$ (d) $-\frac{1}{4x^4}$

Sol. (c) $\int e^{-3\log x}dx = f(x) + C$

$\Rightarrow \int e^{\log x^{-3}}dx = f(x) + C \quad \Rightarrow \int x^{-3}dx = f(x) + C$

$\Rightarrow f(x) + C = \frac{x^{-3+1}}{-3+1} + C$

$\therefore f(x) = -\frac{1}{2x^2}$.

16. The solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$ represents family of

(a) parabolas (b) circles (c) ellipses (d) hyperbolas

Sol. (b) $\frac{dy}{dx} = -\frac{x}{y}$ gives, $\int y\,dy = -\int x\,dx$

$\Rightarrow \frac{y^2}{2} = -\frac{x^2}{2} + C$ i.e., $x^2 + y^2 - 2C = 0$, which represents a family of circles.

21. If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = kA$, then find the value of k.

Sol. $A^2 = AA = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

Now $A^2 = kA$ implies, $\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = k\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix}$

By equality of matrices, we get $2k = 8 \quad \therefore k = 4$.

25. A cylindrical water container has developed a leak at the bottom. The water is leaking at the rate of 5 cm^3/s from the leak. If the radius of the container is 15 cm, find the rate at which the height of water is decreasing inside the container, when the height of water is 2 metres.

Sol. Given $\frac{dV}{dt} = -5 \text{ cm}^3/\text{s}$.

Now $V = \pi r^2 h = \pi \times 15^2 \times h = 225\pi \times h$

$$\Rightarrow \frac{dV}{dt} = 225\pi \times \frac{dh}{dt}$$

$$\Rightarrow -5 = 225\pi \times \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = -\frac{1}{45\pi} \text{ cm/s}$$

Hence, height of water is decreasing at the rate of $\frac{1}{45\pi}$ cm/s.

26. Find $\int \frac{\sqrt{x}}{1+\sqrt{x^{3/2}}} dx$.

Sol. Let $I = \int \frac{\sqrt{x}}{1+\sqrt{x^{3/2}}} dx = \int \frac{\sqrt{x}}{1+x^{3/4}} dx$

Put $x^{3/4} = t \Rightarrow x = t^{4/3} \Rightarrow dx = \frac{4}{3}t^{1/3}dt$. Also, $\sqrt{x} = \sqrt{t^{4/3}} = t^{2/3}$.

$$\text{Now } I = \frac{4}{3}\int \frac{t^{2/3} \times t^{1/3}}{1+t} dt = \frac{4}{3}\int \frac{t}{1+t} dt = \frac{4}{3}\int \left[1 - \frac{1}{1+t}\right] dt$$

$$\Rightarrow I = \frac{4}{3}\left[t - \log|1+t|\right] + C$$

$$\therefore I = \frac{4}{3}\left[x^{3/4} - \log|1+x^{3/4}|\right] + C.$$

29. (a) The probability distribution of a random variable X is given by :

X	0	1	2	3
P(X)	p	$\frac{p}{3}$	$\frac{p}{6}$	$\frac{p}{12}$

(i) Determine the value of p.
(ii) Calculate $P(X \geq 1)$.
(iii) Calculate expectation of X, i.e. $E(X)$.

OR

(b) In a city, a survey was conducted among residents about their preferred mode of commuting. It was found that 50% people preferred using public transport, 35% preferred using a bicycle and 20% use both public transport and a bicycle. If a person is selected at random, find the probability that

(i) the person uses only public transport.
(ii) the person uses a bicycle, given that they also use the public transport.
(iii) the person uses neither public transport nor a bicycle.

Sol. (a) (i) Since $\sum P(X)=1$ so, $p+\frac{p}{3}+\frac{p}{6}+\frac{p}{12}=1$

$\Rightarrow \frac{12p+4p+2p+p}{12}=1$

$\Rightarrow 19p=12$

$\therefore p=\frac{12}{19}$.

(ii) $P(X\geq 1)=1-P(X<1)=1-P(X=0)=1-p=1-\frac{12}{19}=\frac{7}{19}$.

(iii) $E(X)=\sum X.P(X)=0\times p+1\times\frac{p}{3}+2\times\frac{p}{6}+3\times\frac{p}{12}=\frac{2p}{3}+\frac{p}{4}=\frac{11p}{12}=\frac{11}{12}\times\frac{12}{19}=\frac{11}{19}$.

OR

(b) Let A be the event that people used public transport, B be the event that people used bicycle. Given that $P(A)=50\%,\ P(B)=35\%,\ P(A\cap B)=20\%$.

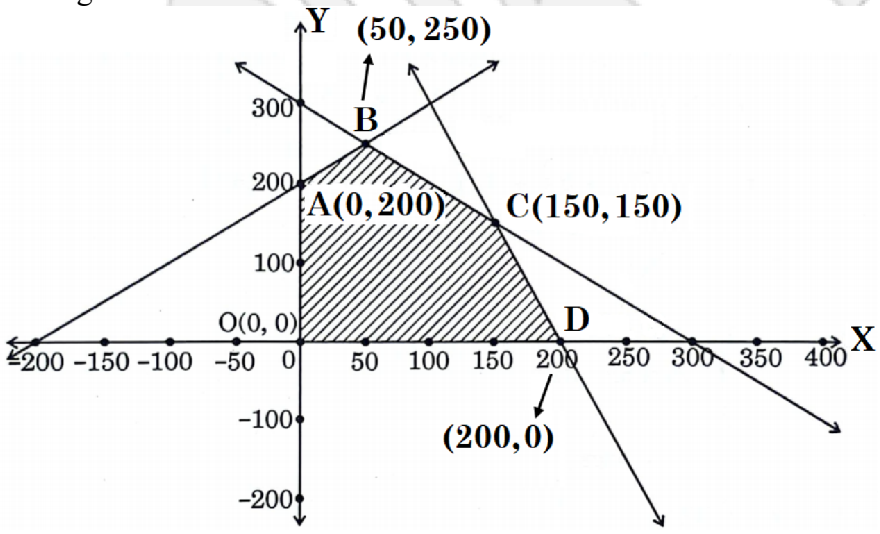
(i) Required probability, $P(\text{only A})=P(A)-P(A\cap B)=50\%-20\%=30\%=\frac{30}{100}=\frac{3}{10}$.

(ii) $P(B\,|\,A)=\frac{P(B\cap A)}{P(A)}=\frac{20\%}{50\%}=\frac{2}{5}$.

(iii) $P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)=1-[P(A)+P(B)-P(A\cap B)]$

$=1-[50\%+35\%-20\%]=1-\frac{65}{100}=\frac{35}{100}=35\%$ or $\frac{7}{20}$.

31. For the given graph of a Linear Programming Problem, write all the constraints satisfying the given feasible region.



Sol. Equation of AB : $\frac{y-200}{x-0}=\frac{0-200}{-200-0}$

$\Rightarrow \frac{y-200}{x}=1 \qquad \Rightarrow x-y=-200$

Equation of line BC : $\frac{y-300}{x-0}=\frac{0-300}{300-0}$

$\Rightarrow \frac{y-300}{x}=-1 \qquad \Rightarrow x+y=300$

Equation of CD : $\dfrac{y-0}{x-200}=\dfrac{150-0}{150-200}$

$\Rightarrow \dfrac{y}{x-200}=-3 \qquad \Rightarrow 3x+y=600$

Since the feasible region is bounded so, the constraints for this LPP will be
$x-y\geq -200,\ x+y\leq 300,\ 3x+y\leq 600,\ x\geq 0,\ y\geq 0$.

34. (a) Evaluate $\int_0^{3/2} |x\cos \pi x|\,dx$.

OR

(b) Find $\int \dfrac{dx}{\sin x+\sin 2x}$.

Sol. (a) Let $I=\int_0^{\frac{3}{2}} |x\cos \pi x|\,dx$

$\Rightarrow I=\int_0^{\frac{1}{2}} |x\cos \pi x|\,dx+\int_{\frac{1}{2}}^{\frac{3}{2}} |x\cos \pi x|\,dx$

$\Rightarrow I=\int_0^{\frac{1}{2}} x\cos \pi x\,dx-\int_{\frac{1}{2}}^{\frac{3}{2}} x\cos \pi x\,dx$

Consider $\int x\cos \pi x\,dx=x\int \cos \pi x\,dx-\int\left(\dfrac{d}{dx}[x]\int \cos \pi x\,dx\right)dx$

$\Rightarrow \quad =x\left(\dfrac{\sin \pi x}{\pi}\right)-\int \dfrac{\sin \pi x}{\pi}dx$

$\Rightarrow \quad =\dfrac{x\sin \pi x}{\pi}+\dfrac{\cos \pi x}{\pi^2}$

Therefore, $I=\left[\dfrac{x\sin \pi x}{\pi}+\dfrac{\cos \pi x}{\pi^2}\right]_0^{\frac{1}{2}}-\left[\dfrac{x\sin \pi x}{\pi}+\dfrac{\cos \pi x}{\pi^2}\right]_{\frac{1}{2}}^{\frac{3}{2}}$

$\Rightarrow I=\left[\dfrac{1}{2\pi}\sin\dfrac{\pi}{2}+\dfrac{1}{\pi^2}\cos\dfrac{\pi}{2}\right]-\left[0+\dfrac{\cos 0}{\pi^2}\right]-\left[\dfrac{3}{2\pi}\sin\dfrac{3\pi}{2}+\dfrac{1}{\pi^2}\cos\dfrac{3\pi}{2}\right]+\left[\dfrac{1}{2\pi}\sin\dfrac{\pi}{2}+\dfrac{1}{\pi^2}\cos\dfrac{\pi}{2}\right]$

$\Rightarrow I=\left[\dfrac{1}{2\pi}+0\right]-\left[\dfrac{1}{\pi^2}\right]-\left[-\dfrac{3}{2\pi}+0\right]+\left[\dfrac{1}{2\pi}+0\right]$

$\therefore I=\dfrac{5\pi-2}{2\pi^2}$.

OR

(b) Let $I=\int \dfrac{dx}{\sin x+\sin 2x}$

$\Rightarrow I=\int \dfrac{dx}{\sin x+2\sin x\cos x}$

$\Rightarrow I=\int \dfrac{dx}{\sin x(1+2\cos x)}$

$$\Rightarrow I=\int\frac{\sin x\ dx}{\sin^2 x(1+2\cos x)}$$

$$\Rightarrow I=\int\frac{\sin x\ dx}{(1-\cos x)(1+\cos x)(1+2\cos x)} \qquad (\text{Put } \cos x=t\Rightarrow \sin x dx=-dt$$

$$\therefore I=\int\frac{-dt}{(1-t)(1+t)(1+2t)}$$

Consider $\frac{-1}{(1-t)(1+t)(1+2t)}=\frac{A}{1-t}+\frac{B}{1+t}+\frac{C}{1+2t}$

$$\Rightarrow -1=A(1+t)(1+2t)+B(1-t)(1+2t)+C(1-t^2)$$

On comparing the coefficients of like terms on both the sides, we get : $A=-\frac{1}{6}, B=\frac{1}{2}, C=-\frac{4}{3}$

$$\therefore I=\int\left(-\frac{1}{6}\times\frac{1}{1-t}+\frac{1}{2}\times\frac{1}{1+t}-\frac{4}{3}\times\frac{1}{1+2t}\right)dt$$

$$\Rightarrow I=-\frac{1}{6}\log|1-t|\times\frac{1}{-1}+\frac{1}{2}\log|1+t|-\frac{4}{3}\log|1+2t|\times\frac{1}{2}+C$$

$$\therefore I=\frac{1}{6}\log|1-\cos x|+\frac{1}{2}\log|1+\cos x|-\frac{2}{3}\log|1+2\cos x|+C\,.$$

With regards,
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